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Oriented slit	Failure criterion	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A theoretical solution is obtained for the growth of narrow plastic zones and for the initiation of ductile fracture at the ends of an angled slit in a metal sheet under remote uniform tension. The theoretical model assumes two distinct regions: narrow plastic zones at the ends of the slit caused by necking of the sheet, and a surrounding elastic region. The direction of the narrow plastic zones is assumed to be normal to the remote tension regardless of the slit angle. The length of the plastic zones is assumed to be governed by Dugdale's hypotheses of yield stress attainment along the plastic

(Continues)

CONTENTS

PLASTIC DEFORMATION AND DUCTILE FRACTURE AT AN ANGLED SLIT IN A SHEET UNDER TENSION

INTRODUCTION

Experimental observations on the failure of uniaxially stretched metal sheets with central slits may, on the basis of sheet thickness, be classified into three types as shown in Fig. 1. For very thin sheets (foils) buckling is observed around the slit, with large out-of-plane elastic deflections; for thicker sheets plastic thinning, or necking, occurs at the ends of the slit; and, for still thicker sheets (plates) heart-shaped plastic zones occur at the ends of the slit with very little thinning.

The present work is restricted to the necking type failure and includes both a theoretical model and its experimental verification. It is clear from the outset that neither a plane stress nor a plane strain model is adequate because of the extensive thinning in the plastic zones. Dugdale [1], however, assumed that the plastic zones could be treated as narrow extensions of the slit with normal traction at the elastic-plastic boundary equal to the yield stress Y, and stress continuity at the ends. He thus reduced the solution in the elastic region to one of plane stress. For the case where the slit is normal to the applied stress, he determined a relationship between the plastic zone length s, and the uniform applied stress T:

$$\frac{s}{a} = \sec\left(\frac{\pi}{2}\frac{T}{Y}\right) - 1, \tag{1}$$

where $2a$ is the original slit length. Using the same model, Goodier and Field [2] and Burdekin and Stone [3] determined a relationship between the opening displacement u_e, at the ends of the slit and the plastic zone length:

$$\frac{u_e}{a} = \frac{8Y}{\pi E}\ln\left(1 + \frac{s}{a}\right), \tag{2}$$

where E is Young's modulus. Arguing that fracture is controlled by in-plane strains, they proposed that slit propagation would occur when s/a reached a critical value. Hahn and Rosenfield [4], however, recognizing the out-of-plane nature of the plastic deformation, proposed that crack propagation would occur when the longitudinal strain ϵ_ℓ at the ends of the slit equalled that at fracture in a standard tensile specimen. They assumed plane strain in the neck with zero strain in the neck direction, and estimated the strain to be given by

$$\epsilon_\ell = \frac{2u_e}{t_i}, \tag{3}$$

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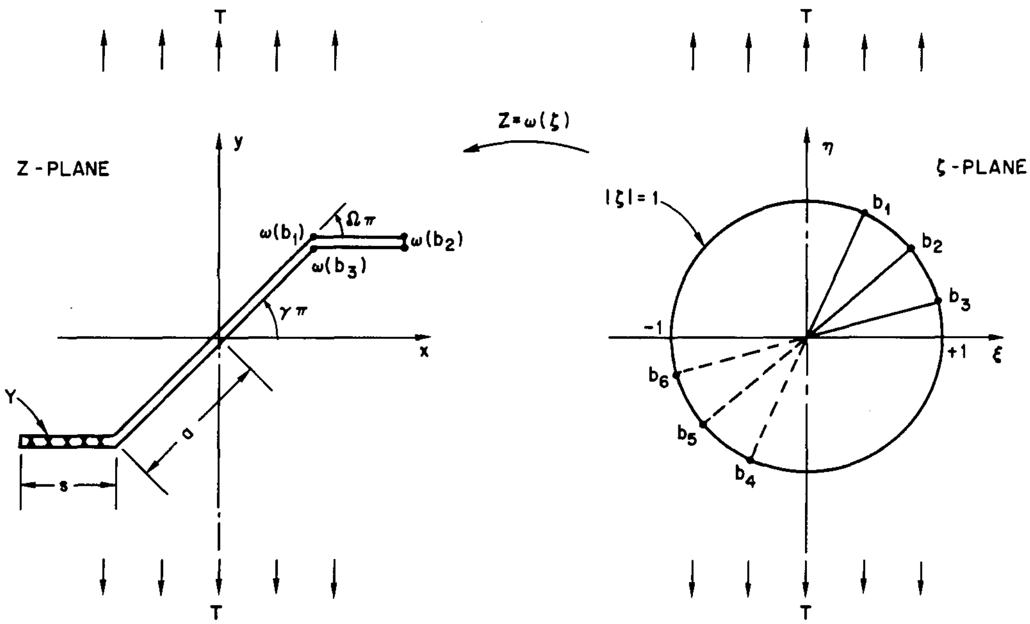

Fig. 2 — Conformal mapping of idealized slit and plastic zone geometry (for $\Omega = \gamma$) onto the unit circle

The out-of-plane deformation within these zones leads to localized necking. After a critical amount of thinning of the sheet takes place, extension of the slit (fracture) begins. The first concern is to justify the assumed direction of the plastic zones at the ends of the slit. The second goal is to determine relationships between the applied traction, the length of the plastic zones, and the slit opening displacements for various slit angles. Finally, a relationship is sought between the opening displacement and the thinning (necking) of the sheet within the plastic zone such that, at a critical value of through-thickness strain at the ends of the slit, fracture initiation may be predicted.

Neck Direction

It is assumed that the plastic zone will grow in the plane of the sheet in the direction normal to the principal tensile stress direction, i.e., $\Omega = \gamma$. This assumption is fully supported by the experimental evidence to be presented below. It is also consistent with a theoretical argument based on analytical solutions. Griffith [9] and later McClintock [10], taking the slit to be the limit of an elliptical hole as the minor axis vanishes, showed that at the end of the slit, this angle is given by

$$\Omega = \gamma/2. \tag{5}$$

Expansion of Eq. (8) results in an infinite series which must be truncated and evaluated on the unit circle. Because this procedure leads to multivalence, the truncated series is evaluated on a circle $1 + \delta$, where $\delta \ll 1$. The magnitude of δ is chosen to preserve univalence, and it is inversely proportional to the number of terms retained in the series. The desired mapping function is

$$\omega(\zeta) = B(\zeta + a_1\zeta^{-1} + a_2\zeta^{-2} + \cdots + a_n\zeta^{-n}), \tag{9}$$

where the a's are complex-valued coefficients and B is a magnification and rotation constant.

The solution is then obtained by finding the proper boundary function $F(\sigma)$ to introduce into the boundary condition

$$\phi(\sigma) + \frac{\omega(\sigma)}{\overline{\omega'(\sigma)}} \overline{\phi'(\sigma)} + \psi(\sigma) = F(\sigma), \tag{10}$$

where $\phi(\sigma)$ and $\psi(\sigma)$ are the values of the stress functions

$$\phi(\zeta) = \sum_{k=1}^{n} c_k \zeta^k \text{ and } \psi(\zeta) = \sum_{k=1}^{n} d_k \zeta^k$$

(which must satisfy the biharmonic equation) at the boundary $\zeta = \sigma = e^{i\upsilon}$.

Thus, when the functions ω, F, ϕ, and ψ are introduced into Eq. (10), equating coefficients of like power terms leads to a set of simultaneous algebraic equations in the unknown c's and d's which, when found, determine the desired stress functions. Then the stresses and displacements can be obtained through the standard formulae [18]:

$$\sigma_{\rho\rho} + \sigma_{\upsilon\upsilon} = 4Re\,\Phi(\zeta) = 2\left[\Phi(\zeta) + \overline{\Phi(\zeta)}\right], \tag{11}$$

$$\sigma_{\upsilon\upsilon} - \sigma_{\rho\rho} + 2i\sigma_{\rho\upsilon} = \frac{2\zeta^2}{\rho^2\overline{\omega'(\zeta)}}\left\{\overline{\omega(\zeta)}\,\Phi'(\zeta) + \omega'(\zeta)\Psi(\zeta)\right\}, \tag{12}$$

and

$$2\mu\,|\omega'(\zeta)|\,(v_\rho + iv_\upsilon) = \frac{\overline{\zeta}}{\rho}\overline{\omega'(\zeta)}\left\{\kappa\phi(\zeta) - \frac{\omega(\zeta)}{\overline{\omega'(\zeta)}}\overline{\phi'(\zeta)} - \overline{\psi(\zeta)}\right\}, \tag{13}$$

where σ^*_J ($J = A, B, C$) are the tangential stresses just outside the tip of the plastic zone due to unit applied tractions at the slit-plastic zone boundary (as calculated from Eqs. (11) and (12)), and the term T arises from the corresponding stress due to the applied traction at infinity.

Thus, from Eq. (14) the dimensionless ratio T/Y is calculated as a function of the dimensionless ratio s/ℓ, where $\ell = a + s$, with the orientation of the slit given by the parameter γ. These results, for several values of γ, are shown in Fig. 3.

To further justify the correctness of the assumed direction for the plastic zone the yield condition (i.e., $0 \leqslant \sigma_{\theta\theta} \leqslant Y$) was systematically verified just outside the boundary of the plastic zone.

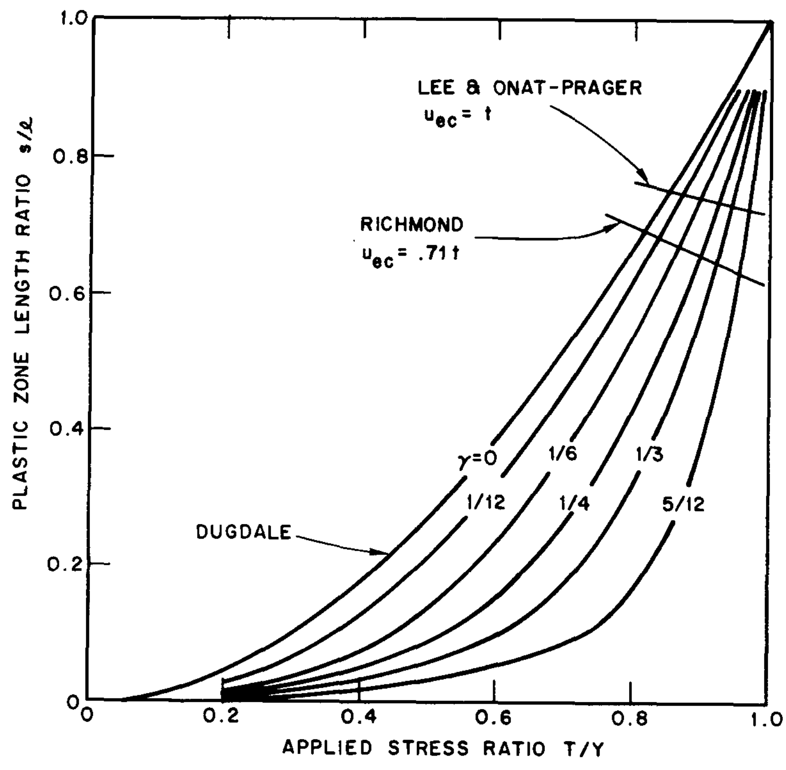

Fig. 3 — Calculated length of plastic zone vs applied stress as a function of slit angle showing upper limit for initiation of slit extension according to different plastic solutions

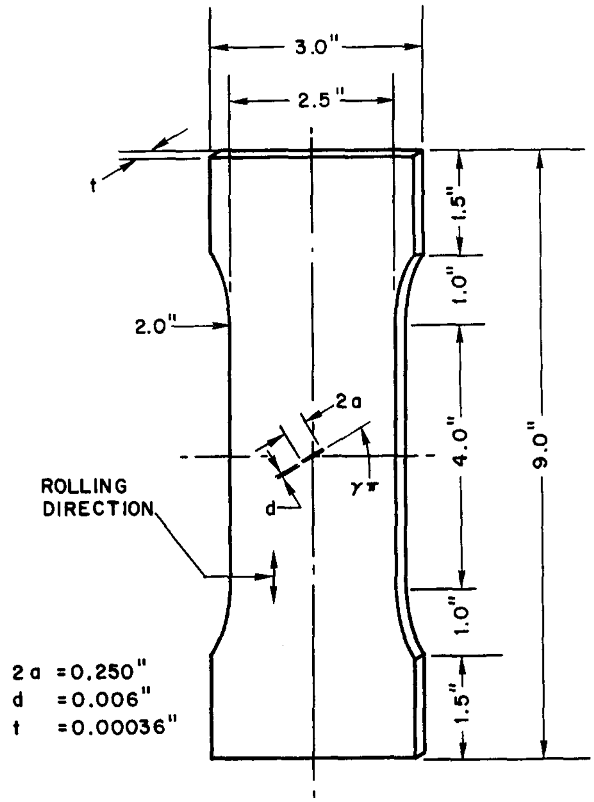

Fig. 4 — Sheet specimen geometry

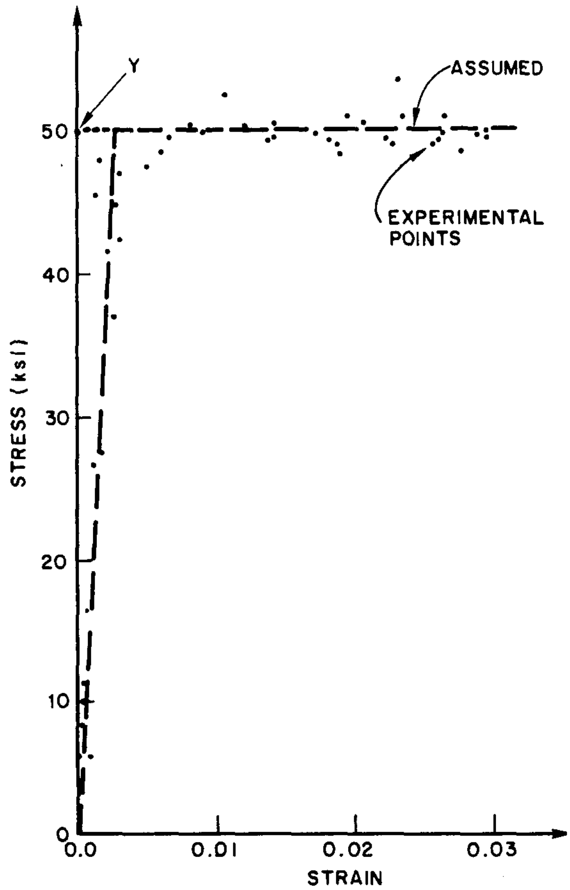

Fig. 5 — Uniaxial stress-strain relationship for hard-rolled copper sheet

The specimen geometry shown in Fig. 4 was chosen by establishing reasonable upper and lower bounds for dimensions and ratios of dimensions that would occur both prior to and during the tests. The pertinent factors were testing facilities, measuring techniques, machining capabilities, isotropy of the test material, prevention of buckling, approximation of boundary conditions, system rigidity, and the need for a large stress concentration.

The ends of the slit are semicircular, with a root radius equal to half the width of the slit, so that, regardless of slit orientation γ, the stress concentration would remain as nearly constant as possible.

The angle between the slit and the perpendicular to the applied traction, given by $\gamma\pi$, was arbitrarily chosen to vary from 0° to 75° in steps of 15° (i.e., $\gamma = 0, 1/12, 1/6, 1/4, 1/3, 5/12$).

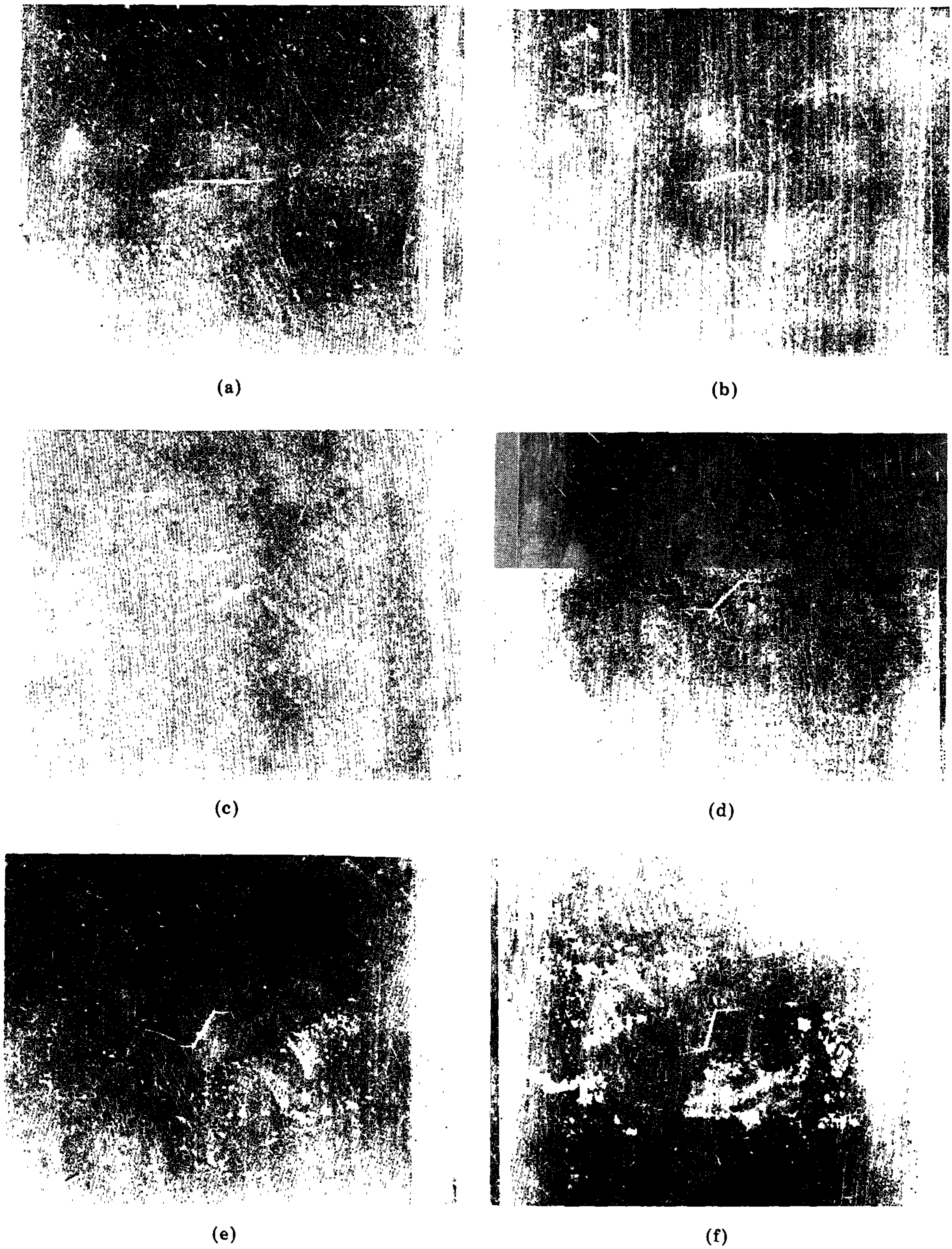

Fig. 6 — Oblique lighting photographs of copper sheet specimens with slits ($\gamma\pi = 0, 15, 30, 45, 60, 75^\circ$) showing the plastic zones caused by uniaxial tensile stress applied in the vertical direction

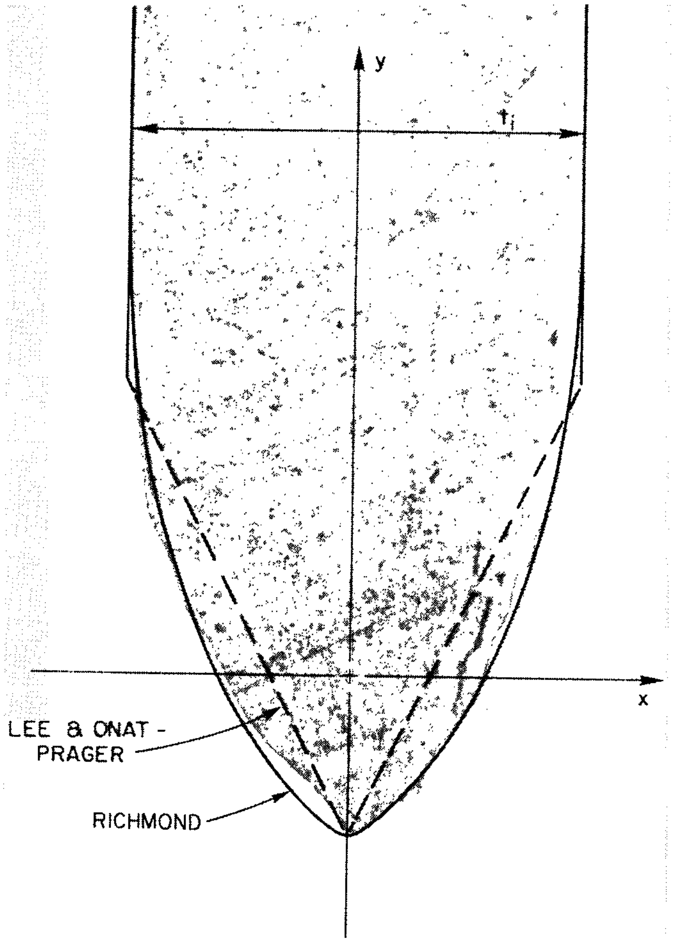

Fig. 8 — Experimental and theoretical neck geometries

In general, the relationship between the applied traction, the plastic zone length, and the slit orientation found experimentally agrees well with the relationship predicted by the theoretical model. In fact, the differences fall within the experimental error up to a T/Y ratio of about 0.7. The discrepancy for T/Y ratios between 0.7 and the upper limit for slit growth initiation may be explained by considering the following four factors: (a) The ratio of average net section stress to yield strength of the material, T_{net}/Y, should not exceed the approximate value of 0.8 for reasons detailed elsewhere [21-23]. When this ratio approaches 0.8, the plastic zone length should be expected to *increase* rapidly. This was indeed observed here. (b) The second factor is buckling of the sheet in the vicinity of the slit, which would lead to plastic zones *shorter* than expected. However, buckling was minimized in these experiments, as explained above. The last two factors also produce opposing effects which could affect the assumption of a constant traction at the elastic-plastic interface. (c) If the material exhibits a small amount of work hardening, the boundary traction would be a function of the amount of strain and the solution would yield *shorter* plastic bands. (d) On the other hand, if the load that results from the necking solution is assumed as the boundary traction, the solution would yield *longer* plastic zones. Either both factors are negligible in

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